FLEXIBLE FORMULA OF ARITHEMTIC

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There are several result on flexible formulae in the literature: formulae such that their extensions as sets are left undetermined by the arithmetical theory in question. In a sense, these formulae can mimic any other formula of a certain complexity. For example, there is a Σ_1-formula $\phi(x)$ such that $\mathbb{N} \models \forall x \neg\phi(x)$, and for each Σ_1-formula $\sigma(x)$, there is an end-extension $\mathcal{N}$ of $\mathbb{N}$ such that $\mathcal{N} \models \forall x(\phi(x) \leftrightarrow \sigma(x))$.

Recently (2011), Hugh Woodin has published a novel generalisation of these results: There is a Σ_1-formula $\psi(x)$ such that for each countable model $\mathcal{M}$ of PA, and for each formula $\delta(x)$ defining a finite extension of $\psi^{\mathcal{M}}$, there is an end-extension $\mathcal{N} \models \mathrm{PA}$ of $\mathcal{M}$ such that $\mathcal{N} \models \forall x(\psi(x) \leftrightarrow \delta(x))$.

In this talk I will present another variation on this theme. Let T be some consistent, r.e. extension of $\mathrm{I}\Sigma_1$. There is a Σ_n-formula $\gamma(x)$ such that for each model $\mathcal{M}$ of $T + \mathrm{Con}(T)$, and for each Σ_n-formula $\delta(x)$, there is an end-extension $\mathcal{N} \models T$ of $\mathcal{M}$ such that $\mathcal{N} \models \forall x(\gamma(x) \leftrightarrow \delta(x))$.