

(IM)PREDICATIVITY AND FREGEAN ARITHMETIC

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In Frege's logicism, numbers are logical objects in the sense that they are extensions of certain concepts. Even though Frege's logical system is inconsistent, it was shown by Richard Heck that its restriction to predicative (second-order) quantification is consistent. This predicative fragment is, nevertheless, too weak to develop arithmetic. This is shown by arguing that the consistency of these theories can be proved in certain subsystems of primitive recursive arithmetic.

Heck's restricted predicative system is, in fact, doubly restricted. Not only is it a restriction on the formation of extensions but also a restriction on the second-order logic. We show that if we enforce the first restriction but give up the second one, then the system is still consistent and is able to develop full second-order arithmetic in a Fregean way. The latter development is made possible because one can prove a form of reducibility that holds of concepts true of only finitely many objects.