JAF 33 Tutorial

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Weak analysis: metamathematics

It is well-known that the provably total functions of the base theory RCA_0 of reverse mathematics are the primitive recursive functions. We show how to set up a similar theory, but one whose provably total functions are (in an appropriate sense) the polytime computable functions. As with the case of RCA_0, one can add to this theory weak König's lemma without proving new Π^0_2-consequences. We draw attention to the pivotal rôle of the bounded collection scheme in these results and to some differences vis-à-vis the usual reverse mathematics setting.

Finally, we briefly describe two other theories of weak analysis: one related to Vaillant's class $\#P$ of counting functions and the other related to polyspace computability.

Weak analysis: mathematics

We briefly describe how to introduce the real numbers in the theory BTFA (base theory for feasible analysis). Continuous functions can also be introduced, following the usual blueprint of reverse mathematics. The intermediate value theorem can be proved and, in particular, the real numbers form a real closed ordered field (but are more than that). We also discuss the rôle of weak König's lemma in the setting of BTFA.

However, Riemann integration for continuous functions with a modulus of uniform continuity cannot be developed in BTFA: the theory for doing Riemann integration must be able to do some counting. With enough counting, one has a decent theory of integration up to the fundamental theorem of calculus. Finally, we pose some open questions. For instance, what is the strength of Weierstrass approximation theorem in the weak analysis setting?