A MODEL WITH NO COLLECTION AND NO EXPONENTIATION

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I plan to discuss some results obtained in the last few years on the old Wilkie-Paris problem whether there exists a model of $I\Delta_0 + \neg\exp + \neg B\Sigma_1$.

A simple theorem published in a joint paper with Zofia Adamowicz and Jeff Paris states that to get a model of $I\Delta_0 + \neg\exp + \neg B\Sigma_1$ it suffices to have a (reasonable) model of $I\Delta_0 + \neg\exp$ with a Σ_1 truth definition for Σ_1 sentences (not for general Σ_1 formulas). This seems like progress, because, as shown in the same paper, for each $n > 1$ it is possible to have a Σ_n truth definition for Σ_n sentences in a model without exponentiation. Nothing analogous is known for general Σ_n formulas, so the case of sentences does appear to be genuinely easier.

However, some new results suggest that obtaining the Σ_1 truth definition for Σ_1 sentences without exponentiation may be quite difficult. Firstly, there is a complexity-theoretic statement, of the "very unlikely but potentially very hard to disprove" variety, which implies that $B\Sigma_1$ actually follows from $\neg\exp$, over Π_1-truth or over plain $I\Delta_0$ depending on whether the statement is just true or provable in $I\Delta_0$. Secondly, a proof of the unprovability of any fixed finite fragment of $B\Sigma_1$ from $\neg\exp$ would have to be "non-relativizing".