LOCAL INDUCTION AXIOMS VS LOCAL INDUCTION RULES

A. Cordón–Franco and F.F. Lara–Martín

Dpto. Ciencias de la Computación e Inteligencia Artificial
Facultad de Matemáticas. Universidad de Sevilla
C/ Tarfia, s/n, 41012 Sevilla (Spain)
{acordon,fflara}@us.es

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Abstract

Mints, Adamowicz–Bigorajska, Kaye and Ratajczyk (independently) proved that if a Π_2–sentence θ is derived (over the base theory $I\Delta_0$) using m instances of parameter–free Σ_1–induction axiom scheme then θ can also be derived using at most m (nested) applications of Σ_1–induction rule. A similar result holds when Σ_1–induction scheme is replaced with a *local* version of the induction principle, namely, the following scheme $I(\Sigma_1^-, \mathcal{K}_1)$:

$$\varphi(0) \wedge \forall x\,(\varphi(x) \rightarrow \varphi(x+1)) \rightarrow \forall x \in \mathcal{K}_1\, \varphi(x)$$

where $\varphi(x)$ is a parameter–free Σ_1 formula and $\forall x \in \mathcal{K}_1\, \varphi(x)$ expresses that every Σ_1-definable element satisfies $\varphi(x)$. In this talk, working over $I\Delta_0$, we obtain new conservation results relating the number of instances of $I(\Sigma_1^-, \mathcal{K}_1)$ needed to derive a sentence θ, and the number and depth of *nested* applications of several induction rules needed in a derivation of θ. Several formulations of induction rules are considered in correspondence with the quantifier complexity of the sentence θ (Π_2, $\mathcal{B}(\Sigma_1)$ or Π_1). Since $I(\Sigma_1^-, \mathcal{K}_1)$ and the parameter–free Π_1–induction scheme, $I\Pi_1^-$, are equivalent over $I\Delta_0$, we shall derive as corollaries some new conservation results for this last scheme.

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