

Use of nonstandard models in reverse mathematics

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In the field of reverse mathematics, many proof techniques and separation techniques are imported from the computability theory, and hence, reverse mathematical results are often understood over ω -models. (For the general study of reverse mathematics, see Simpson[3].) On the other hand, in the recent study of reverse mathematics, use of nonstandard models is getting more and more important. Actually, several different types of applications of nonstandard methods are known. In this talk, I will show several examples of those methods, and give a brief introduction of applications of nonstandard methods to reverse mathematics. One specific method is a hybrid of techniques from computability and nonstandard models. Sometimes, a computability theoretic method works over a nonstandard model as usual with a careful study, and sometimes an irregular computable method is available on a certain special nonstandard model. Chong/Yang/Slaman[1] showed that the stable Ramsey's theorem for pairs does not imply the full Ramsey's theorem for pairs using this method. Another important example is an application of nonstandard analysis to reverse mathematics through the construction of end extensions. This method is first introduced by Tanaka[4] using self-embedding theorem for WKL_0 [5], and now it leads even reverse mathematics for nonstandard analysis itself, e.g., [2]. I will also give some other recent examples: one is a hybrid of randomness and nonstandard models, and another one is a nonstandard methods for reverse mathematics with a weaker base theory. The latter one is a joint work with Kołodziejczyk.

References

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